973 项目2013CB834202 – 2014年3月8日-9日总结会摘要

贾朝华：关于除数函数

除数函数是数论中的一个重要函数，关于它的研究有很长的历史。本报告将介绍除数函数的一些性质、结果以及一个最新的进展。

王崧： Grunwald-Wang Theorem and S-version of the Strong Multiplicity One

Grunwald-Wang is a famous classical theorem in class field theory. In this talk we will sketch the proof of the effective version, and more over explain the strong S-multiplicity one which is used in the proof plus several variations.

魏达盛： The Unramified Brauer Group of Norm One Tori

Let k be a number field and K/k Galois. We transform the construction of the unramified Brauer group of the norm one torus $R^1_{K/k}(\mathbf{G}_m)$ into the construction of a special abelian extension over K. If $k = \mathbb{Q}$ and $K/\mathbb{Q}$ biquadratic, we explicitly construct the unramified Brauer group of $R^1_{K/\mathbb{Q}}(\mathbf{G}_m)$.

吕鑫：

张翀：关于自守周期

我将介绍一下自守周期当前的发展及常用工具相对迹公式，并且汇报一下我所做的工作。

蔡立：精确 Gross-Zagier 公式与 Waldspurger 公式

Gross-Zagier 公式与 Waldspurger 公式在数论问题的研究上有重要的意义。基于袁-张-张的工作，我们最近得到了精确版本的 Gross-Zagier 公式与Waldspurger 公式。我们得到的公式是前人所希望的并覆盖现在已有的类似结果。这是与田野教授和舒杰合作完成的。

康云凌：关于 p-adic L-function 的 mu 不变量的计算

本报告将简单介绍一种计算 p-adic L-function 的 mu 不变量的初等方法。

陈珂： Special subvarieties in Kuga varieties

We study the Andre-Oort conjecture for Kuga varieties, which are universal family of abelian varieties over Shimura varieties. We define the notion of rigid special sub-varieties of Kuga varieties, and show that the Zariski closure of a sequence of rigid special sub-varieties with uniformly bounded Galois orbits can be represented as a finite union of rigid special sub-varieties.

许宾： Fourier coefficients of automorphic forms and the Gross-Prasad conjecture for special orthogonal groups

In this talk, we consider a special type of Fourier coefficients of some kind of residual representations, and study its connection to the weak Gross-Prasad conjecture for special orthogonal groups.

陆晴： 8-rank of class groups and isotropy index

Suppose $F = \mathbb{Q}(\sqrt{-p_1 \cdots p_t})$ is an imaginary quadratic number field with distinct primes $p_1, \ldots, p_t$, where $p_i \equiv 1 \pmod 4 (i = 1, \ldots, t-1)$ and $p_t \equiv 3 \pmod 4$. We express the possible values of the 8-rank r_8 of the class group of F in terms of a quadratic form Q over $\mathbb{F}_2$ which is defined by quartic symbols. In particular, we show that r_8 is bounded by the isotropy index of Q.

刘余： The construction of a p-adic L-function

In this talk, I will use a norm compatible system of Gross point in a definite Shimura set, to construct a p-adic L function associated to a modular form of weight 2, via the periods.

李永雄： On Results of Quadratic Twists of $X_0(49)$

In this talk, we report some results on the quadratic twist of the elliptic curve $E = X_0(49)$. First, we introduce an infinite family of rank zero quadratic twist of E satisfying the B-SD conjecture. Second, we give a result which is a generalization of Birch's lemma to an infnitely family of rank one quadratic twist using Kolyvagin's Euler system method. Finally, we give the main theorem on the rank one quadratic twist of the curves belong to the family in the second step, which use Tian's induction argument on Heegner points. If time permit, We will talk about some recent work on a more general setting of the above main theorem, done by the forthcoming paper of Cai-Shu-Tian.

王章结： Cassels pairing and congruent elliptic curves

It is a very old problem to determine whether a positive number is the area of a rational right triangle, which is equivalent to there is an infinity order point on $E^{(n)} : y^2 = x^3 - n^2x$. Fermat first proved 1 is a non-congruent number using his infinity descent. Mordell generalized this to show that the $\mathbb{Q}$-points of any elliptic curve over $\mathbb{Q}$ form a finitely generated abelian group.

Descent then was widely used to study elliptic curves, especially the non-congruent elliptic curves. Feng and many others studied some family non-congruent numbers with 2-primary part of the Shafarevich-Tate group trivial via 2-isogeny. Li and Tian found a sufficient condition for n to be a non-congruent number and 2-primary part of Sha non-trivial, if n has all prime factors congruent to 1 (mod 8). Then Ouyang and Zhang also found a sufficient condition for n to be non-congruent and 2-primary part of Sha non-trivial with n having all prime factors congruent to 1 (mod 4). In this talk, I will talk about Cassels pairing, and using this pairing I will characterize when n to be non-congruent with 2-primary part of Sha isomorphic to $(\mathbb{Z}/2\mathbb{Z})^2$ with 8-rank of ideal class groups, if $n \equiv 1(\mod 8)$ with all prime factors equivalent to 1 (mod 4). For $n \equiv 3(\mod 8)$, we have similar results. While for $n \equiv 5, 7(\mod 8)$, we can also characterize when the Cassels pairing on Sel_2 non-trivial.

舒杰： Cubic Sum Problem

I will talk about the joint work with Ye Tian and Li Cai on cubic twists of the elliptic curve $y^2 = x^3 + 1$. The main result is that, for any integer $k \geq 1$, there exist infinitely many cube-free natrual numbers with exactly k

odd prime factors such that the Diophatine equation $2n = x^3 + y^3$ is soluble in rational numbers.

张神星：Waldspurger formula over function fields

With the assumptions of Siegel-Weil formula and Tunnell-Saito theorem in function field case, I will give an analog of Waldspurger formula over function fields, which gives a relation between the periods and special value of L-function。

杨金榜：On the cohomology of semi-stable p-adic Galois representations

I will give a simple proof of Hyodo's celebrated result $H_g^1(K, V) = H_{st}^1(K, V)$ for V a potentially semi-stable Galois representation.

张哲：双二次域的希尔伯特亏格域 (Hilbert genus fields of biquadratic fields)